EXERCISES [MAI 1.6]

GEOMETRIC SEQUENCES

SOLUTIONS

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A. Paper 1 questions (SHORT)

1. (a) $u_1 = 10, \quad r = 2$

(b) 5120

(c) 10230

(d) $10\times 2^{n-1}$ $(= 5\times 2^n)$

(e) $n = 12$

2. (a) $u_1 = 10, \quad r = 0.5$

(b) 0.0195

(c) 19.98

(d) $10\times 0.5^{n-1}$ $(= 20\times 0.5^n)$

(e) $n = 6$

3. (a) $\frac{54}{18}=\frac{162}{54}=\frac{486}{162}$ $(=3)$

(b) (i) $r = 3$

$u_n = 18 \times 3^{n-1}$

(ii) $18 \times 3^{n-1} = 1062882$

$n = 11$

4. (a) 6, 12, 24

(b) $\sum_{n=1}^{3}\left(3\times 2^n\right) = 6 + 12 + 24 = 42$

(c) $\sum_{n=1}^{12}\left(3\times 2^n\right)$=24570.

5. (a) $\frac{a}{8}=\frac{1}{2} \Leftrightarrow a = 4$

OR

$\frac{2}{a}=\frac{1}{2} \Leftrightarrow a = 4$

(b) $8\left(\frac{1}{2}\right)^7 = 0.0625$

(c) $\dfrac{8\left(\left(\frac{1}{2}\right)^{12}-1\right)}{\frac{1}{2}-1} = 16.0 (3\ s.f)\quad (=4095/256)$

6. (a) $0.5 \left(\frac{1}{2}\right)$

(b) (i) $a = 4$ (ii) $b = 1$

(c) $\frac{16(1-0.5^n)}{(1-0.5)} = 31.9375$

$n = 9$

7. (a) $28 = 7r^2$
$r = 2$

(b) 114681

8. (a) $r = \frac{36}{108}\left(\frac{1}{3}\right)$

(b) $u_1\left(\frac{1}{3}\right)^7 = 36$

$u_1 = 78732$

(c) $118096 = \frac{78732\left(1-\left(\frac{1}{3}\right)^k\right)}{\left(1-\frac{1}{3}\right)}$

$k = 10$

9. (a) $-\frac{1800}{3000} = -0.6$

(b) $u_{10} = 3000(-0.6)^9 = -30.2$ (accept the exact value -30.233088)

(c) $S = 1863.66$

10. $2 \times 1.05^{n-1} > 500$ so $1.05^{n-1} > 250$

METHOD A: Trial and error;

The smallest integer that satisfies the inequality is $n = 115$. Then $u_{115} = 521$

METHOD B: By using GDC SolveN or Graphical solution

The smallest integer that satisfies the inequality is $n = 115$. Then $u_{115} = 521$

METHOD C: Using logarithms;

$n - 1 > \frac{\log 250}{\log 1.05} \Leftrightarrow n - 1 > 113.1675...$ *so* $n = 115$. Then $u_{115} = 521$

11. (a) 2 (b) 80 (c) 5115 (d) 9 (e) 1280

12. (a) 0.5 (b) 31.25 (c) 999.023 (d) 7 (e) 7.8125

13. (a) $u_4 = u_1 r^3 \Leftrightarrow \frac{1}{81} r^3 = \frac{1}{3} \Leftrightarrow r = 3$

(b) $\frac{\frac{1}{81}(3^n - 1)}{2} > 40; \Leftrightarrow n > 7.9888...$ So $n = 8$

(c) if $n = 7$, $\mathbf{S_7} = 13.49...$; if $n = 8$, $\mathbf{S_8} = 40.49...$ which is > 40

14. (a) $u_1 r^4 = 324 \Leftrightarrow u_1 r = 12 \Leftrightarrow r^3 = 27 \Leftrightarrow r = 3$

(b) $4 \times 3^9 = 78732$ **or** $12 \times 3^8 = 78732$

(c) $4 \times 3^{k-1} > 2000$

$k > 6$, So $k = 7$

15. (a) $r = 3$, $u_1 = 5$

(b) $r = \frac{1}{3}$, $u_1 = 2657205$

(c) $r = 3$, $u_1 = 5$ OR $r = -3$, $u_1 = -5$

16. (a) $\frac{x}{5} = \frac{45}{x} \Leftrightarrow x^2 = 225 \Leftrightarrow x = 15$ or $x = -15$

(b) if $x = 15$, then $y = 135$, if $x = -15$, then $y = -135$

17. (a) $2k - k = k + 60 - 2k \Leftrightarrow 2k = 60 \Leftrightarrow k = 30$

(b) $\frac{2k}{k} = \frac{k+60}{2k} \Leftrightarrow 2 = \frac{k+60}{2k} \Leftrightarrow 4k = k + 60 \Leftrightarrow 3k = 60 \Leftrightarrow k = 20$

(c) For $k = 30$ the sequence is 30, 60, 90 (arithmetic with $d = 30$)

For $k = 20$ the sequence is 20, 40, 80 (geometric with $r = 2$)

18. $1 - a = b - 1$ and $b = a^2 \Leftrightarrow a^2 + a - 2 = 0 \Leftrightarrow a = -2, b = 4$

19. (a) $u_{11} = u_1 + 10d \Leftrightarrow -16 + 10d = 39 \Leftrightarrow 10d = 55 \Leftrightarrow d = 5.5$

(b) $u_3 = u_1 r^2 \Leftrightarrow u_1 r^2 = 12$

$u_5 = u_1 r^4 \Leftrightarrow u_1 r^4 = \frac{16}{3}$

$r^2 = \frac{\left(\frac{16}{3}\right)}{12} = \frac{16}{36} = \frac{4}{9} \Leftrightarrow r = \frac{2}{3}$

20. (a) $u_{96} = u_1 + 95d = 0 + 95 \times 12 = 1140$

(b) $6r^5 = 16d \Leftrightarrow 6r^5 = 16 \times 12 \Leftrightarrow 6r^5 = 192$

(c) $r^5 = 32 \Leftrightarrow r = 2$

(d) $0 + (n-1) \times 12 = 6 \times 2^{n-1} \Leftrightarrow n = 2$ or $n = 3$

(Indeed, the 2^{nd} term of each sequence is 12, the 3^{rd} term of each sequence is 24)

PROBLEMS

21. (a) $r = \frac{8320}{8000} \Leftrightarrow r = 1.04$

(b) Fees = $8000\,(1.04)^6 = 10122.55$ USD (USD not required)

(c) Total = $\frac{8000(1.04^8 - 1)}{1.04 - 1} = 73713.81$ USD (USD not required)

Financial penalty (FP) for no 2 dp applies in parts (b) and (c)

22. (a) (i) 2 minutes + 6 seconds + 6 seconds = 2 minutes 12 seconds (or 2.2 minutes)

(ii) $2(1.05)^2 = 2.205$

(b) $S_{10} = \frac{2(1.05^{10} - 1)}{(1.05 - 1)} = 25.2$ minutes (or 25 minutes 12 seconds)

(c) the common difference for John is 6 seconds = 0.1 minutes

$S_{10} = \frac{10}{2}(2 \times 2 + 9 \times 0.1) = 24.5$ minutes

(or 24 minutes 30 seconds)

23. (a) $r = \frac{2500}{2000} = 1.25$

(b) $S_6 = \frac{2000(1.25^6 - 1)}{1.25 - 1} = 22517.57813........ = 22518$ (to the nearest dollar)

24. (a) Let the population at the end of 1999 be x.

$\frac{44100}{x} = \frac{x}{40000} \Leftrightarrow x = 42\,000$

(b) $r = \frac{44100}{42000} = 1.05$

$u_n = u_1 r^{n-1}$

METHOD A

Assume that u_1 if for 1992 and $u_5 = 40\,000$ is for 1996

$40\,000 = u_1(1.05)^4$

$u_1 = 32\,908$ (or 32 900 to 3 s.f.)

METHOD B

For 4 years before 1996 we **divide** 40 000 by $(1.05)^4$

$\frac{40000}{(1.05)^4} = 32908$

B. Paper 2 questions (LONG)

25. (a) $r = \frac{360}{240} = \frac{240}{160} = \frac{3}{2} = 1.5$

(b) 2002 is the 13^{th} year.
$u_{13} = 160(1.5)^{13-1} = 20759$ (Accept 20760 or 20800.)

(c) $5000 = 160(1.5)^{n-1}$
$\frac{5000}{160} = (1.5)^{n-1} \Rightarrow n = 9.49 \Rightarrow 10^{th}$ year $\Rightarrow 1999$

OR

Using a gdc recursion with $u_1 = 160$, $u_{k+1} = \frac{3}{2}u_k$, $u_9 = 4100$, $u_{10} = 6150$ So 1999

(d) $S_{13} = 160\left[\frac{1.5^{13} - 1}{1.5 - 1}\right] = 61958$ (Accept 61960 or 62 000)

(e) Nearly everyone would have bought a portable telephone so there would be fewer people left wanting to buy one.

OR Sales would saturate.

26. (a) (i) $r = -2$

(ii) $u_{15} = -3\,(-2)^{14} = -49152$ (accept -49200)

(b) (i) 2, 6, 18

(ii) $r = 3$

(c) $\frac{x+1}{x-3} = \frac{2x+8}{x+1} \Leftrightarrow x^2 + 2x + 1 = 2x^2 + 2x - 24 \Leftrightarrow x^2 = 25 \Leftrightarrow x = 5$ or $x = -5$

So $x = -5$

(d) (i) $-8, -4, -2$

(ii) $r = \frac{1}{2}$

27. (a) $\frac{1-3^n}{1-3} = 29\,524 \Leftrightarrow n = 10.$

(b) Common ratio is $\frac{1}{3}$, $(0.333\,(3s.f.)$

(c) $\frac{1-\left(\frac{1}{3}\right)^{10}}{1-\frac{1}{3}} = 1.50$ (3s.f.)

(d) Both $\left(\frac{1}{3}\right)^{10}$ and $\left(\frac{1}{3}\right)^{1000}$ (or those numbers divided by 2/3) are 0 when corrected to 3s.f., so they make no difference to the final answer.

Notes: Accept any valid explanation

(e) The sequence given is $G_1 + G_2$

The sum is 29 524 + 1.50 = 29 525.5

28. (a) $1024r^3 = 128 \Leftrightarrow r^3 = \frac{1}{8} \Leftrightarrow r = \frac{1}{2} = 0.5$

(b) $1024 \times 0.5^{10} = 1$

(c) $$S_8 = \frac{1024\left(1-\left(\frac{1}{2}\right)^8\right)}{1-\frac{1}{2}} = 2040$$

(d) $$\frac{1024\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}} > 2047.968$$

$n = 16$

(e) $S_{15} = 2047.9375$
$S_{16} = 2047.96875$

So $n = 16$

29. (a) (i) $T_1 = \$250$
$d = \$200$
$T_{10} = 250 + 9 \times 200 = 2050$

(ii) $T_1 = \$10$
$r = 2$
$T_{10} = 10 \times 2^9 = 5120$

(b) $S_{10} = \frac{10}{2}(250 + 2050) = 11500$ **OR** $S_{10} = \frac{10}{2}\{2 \times 250 + (10 - 1) \times 200\} = 11500$

(c) Option One: \$10000
Option Two: \$11500

Option Three: $S_{10} = \frac{10(2^{10}-1)}{2-1} = 10\ 230$

Therefore, Option Two would be best.

30. (a) $u_1 = 135 + 7(1) = 142$

(b) $u_2 = 135 + 7(2) = 149$
$d = 149 - 142 = 7$ (**OR** *alternatives*)

(c) $S_n = \frac{n[2(142)+7(n-1)]}{2} = \frac{n[277+7n]}{2} = \frac{7n^2}{2} + \frac{277n}{2}$ $(= 3.5n^2 + 138.5n)$

(d) $20r^3 = 67.5 \Leftrightarrow r^3 = 3.375 \Leftrightarrow r = 1.5$

(e) $T_7 = \frac{20(1.5^7-1)}{(1.5-1)} = 643$ *(accept 643.4375)*

(f) $$\frac{20(1.5^n-1)}{(1.5-1)} > \frac{7n^2}{2} + \frac{277n}{2}$$

$n = 10$